Math 10A
Worksheet, Discussion #18; Wednesday, 7/11/2018
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1 Symmetry

1.1 Concepts

1. A function is called **even** if $f(x) = f(-x)$ and **odd** if $f(x) = -f(-x)$. For an even function $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$. For an odd function, $\int_{-a}^{a} f(x)dx = 0$.

1.2 Problems

2. **TRUE** False An even function is symmetric across the y axis.

3. True **FALSE** An odd function is symmetric across the x axis.

4. Is $f(x) = x^3 + x$ even, odd, or neither?

Solution: Plugging in $-x$ we get $(-x)^3 + (-x) = -x^3 - x = -(x^3 + x) = -f(x)$ so the function is odd.

5. Is $f(x) = \sqrt{1 - x^4}$ even, odd, or neither?

Solution: $f(-x) = \sqrt{1 - (-x)^4} = \sqrt{1 - x^4} = f(x)$ so even.

6. Is $f(x) = x^5 + x^2$ even, odd, or neither?

Solution: $f(-x) = (-x)^5 + (-x)^2 = -x^5 + x^2$ so neither.

7. Is $f(x) = \frac{x}{x^2 + 1}$ even, odd, or neither?

Solution: $f(-x) = \frac{-x}{(-x)^2 + 1} = \frac{-x}{x^2 + 1} = -f(x)$ so odd.

2 Integration by Parts

8. True **FALSE** Integration by parts will automatically give the antiderivative of a function.

9. Find $\int \arctan(x)dx$.

Solution: For choosing a u, we should look for logarithms and inverse trig functions first. Then, we should look for polynomials, then trig functions and finally exponential functions. So, we set $u = \arctan(x)$ and set dv as the rest or $dv = 1dx$. Thus, we have that

$$\int \arctan(x)dx = uv - \int vdu = x\arctan(x) - \int \frac{x}{1+x^2}dx.$$

We can solve the last one with a u substitution $u = 1+x^2$ so $du = 2xdx$ and $xdx = \frac{du}{2}$ so

$$\int \arctan(x)dx = x\arctan x - \frac{1}{2}\int \frac{du}{u} = x\arctan x - \frac{\ln(1+x^2)}{2} + C.$$

10. Find $\int \sin(x)\cos(x)dx$.

Solution: We can use u substitution but also use integration by parts. Let $u = \sin(x)$ and $dv = \cos(x)dx$ so $v = \sin(x)$. Thus

$$\int \sin(x)\cos(x)dx = \sin^2(x) - \int \cos(x)\sin(x)dx.$$

So we have that

$$2\int \sin(x)\cos(x)dx = \sin^2(x) + C \implies \int \sin(x)\cos(x)dx = \frac{\sin^2(x)}{2} + C.$$

11. Integrate $\int x\ln x dx$.

Solution: Let $u = \ln x$ and $dv = xdx$ so

$$\int x\ln x dx = \frac{x^2\ln x}{2} - \int \frac{x^2}{2}\frac{1}{x}dx = \frac{x^2\ln x}{2} - \int \frac{x}{2}dx = \frac{x^2\ln x}{2} - \frac{x^2}{4} + C.$$

12. Integrate $\displaystyle\int \frac{\ln x}{x^5} dx$.

Solution: Let $u = \ln x$ and $dv = x^{-5} dx$ so $v = \frac{-x^{-4}}{4}$ and so

$$\int \frac{\ln x}{x^5} dx = \frac{-\ln x}{4x^4} - \int \frac{-1}{4x^4} \cdot \frac{1}{x} dx = \frac{-\ln x}{4x^4} - \int \frac{-1}{4x^5} dx = \frac{-\ln x}{4x^4} - \frac{1}{16x^4} + C.$$

13. Integrate $\displaystyle\int (\ln x)^2 dx$.

Solution: Let $u = (\ln x)^2 dx$ and $dv = 1dx$ so $v = x$. Then $du = \frac{2\ln x}{x} dx$ and

$$\int (\ln x)^2 dx = x(\ln x)^2 - \int \frac{2x \ln x}{x} dx = x(\ln x)^2 - 2\int \ln x dx$$

$$= x(\ln x)^2 - 2\left[x \ln x - \int x/x dx\right] = x(\ln x)^2 - 2x\ln x + 2x + C.$$

14. Integrate $\displaystyle\int x(\sin x + \cos x) dx$.

Solution: Let $u = x$ and $dv = (\sin(x) + \cos(x))dx$ so $du = dx$ and $v = -\cos(x) + \sin(x)$. So

$$\int x(\sin(x) + \cos(x))dx = x(\sin(x) - \cos(x)) - \int \sin(x) - \cos(x) dx$$

$$= x(\sin(x) - \cos(x)) - (-\cos(x) - \sin(x)) + C.$$

15. Integrate $\displaystyle\int_{\tan(1)}^{\tan(e)} \frac{\ln(\arctan(x))}{1+x^2} dx$.

Solution: u sub with $u = \arctan(x)$ so $du = \frac{dx}{1+x^2}$ and our integral becomes

$$\int_1^e \ln(u) du = u\ln(u) \mid_1^e - \int_1^e du = e\ln e - 1\ln 1 - (e-1) = e\ln e - e + 1 = 1.$$

16. Integrate $\int_{\pi/4}^{\arctan(e)} \sec^2(x)\ln(\tan(x))dx.$.

Solution: u sub with $u = \tan(x)$ and $du = \sec^2(x)dx$ to get

$$\int_1^e \ln(u)du = e\ln e - e + 1 = 1.$$